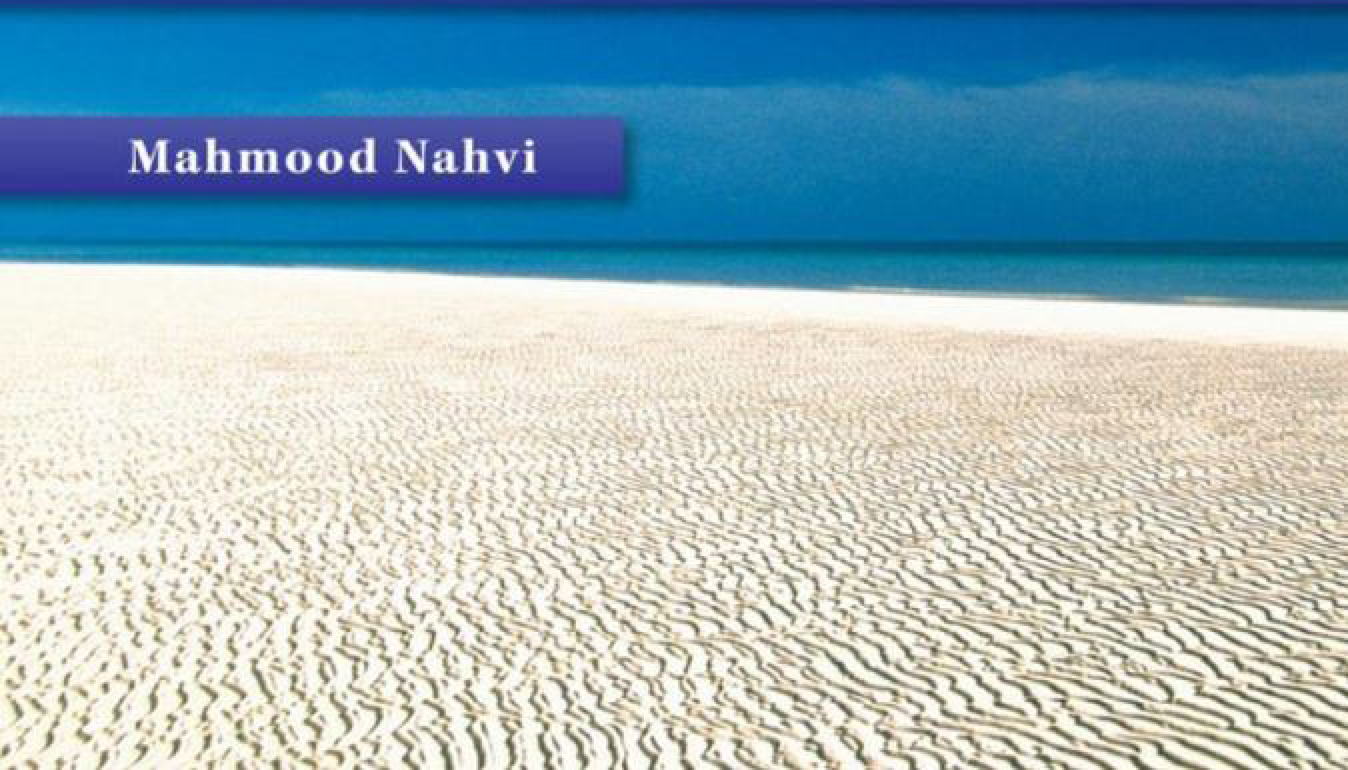
Signals *and* Systems

Mahmood Nahvi



Signals and Systems

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SIGNALS AND SYSTEMS

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BRIEF CONTENTS

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PREFACE

The subject of signals and systems is a requirement in undergraduate electrical and computer engineering programs. The subject provides the student a window through which he or she can look into and examine the field. In addition, it provides the necessary background for more specialized subjects, including communication, control, and signal processing. Several other engineering majors offer similar courses in the same subject matter.

This book is designed to serve as the primary textbook for a course in signals and systems at the junior undergraduate level. It is to be used mainly in the electrical, electronics, and computer engineering programs but is also appropriate for other engineering majors. It may be used in a one- or two-semester or two-quarter sequence according to the criteria of the curriculum and depending on an appropriate selection of material which meets the needs and backgrounds of students.

This book treats the continuous- and discrete-time domains separately in two parts. Part One (Chapters 1–9) covers continuous-time signals and systems; Part Two (Chapters 10–18) covers discrete-time signals and systems. Both parts stand alone and can be used independently of each other. This allows instructors to use the text for instruction on either domain separately, if desired. The book may also be used for courses that teach the two domains simultaneously in an integrated way, as the chapters in Parts One and Two provide parallel presentations of each subject. The parallelism of the chapters on the continuous- and discrete-time domains facilitates the integration of the two parts and allows for flexibility of use in various curricula. The chapter topics and the parallelism between the time-domain treatments are listed in the table below.

Part One, Continuous-Time Domain		Part Two, Discrete-Time Domain	
Chapter	**Topic**	**Chapter**	**Topic**
1	Introduction to Signals	10	Time-Domain Sampling and Reconstruction
2	Sinusoids	11	Discrete-Time Signals
3	Systems, Linearity, and Time Invariance	12	Linear Time-Invariant Discrete-Time Systems
4	Superposition, Convolution, and Correlation	13	Discrete Convolution
5	Differential Equations and LTI Systems	14	LTI Difference Equations
6	The Laplace Transform and Its Applications	15	The z-Transform and Its Applications
7	Fourier Series	16	Discrete-Time Fourier Transform
8	Fourier Transform	17	Discrete Fourier Transforms
9	System Function, the Frequency Response, and Analog Filters	18	System Function, the Frequency Response, and Digital Filters

Whether the subject of signals and systems in the continuous- and discrete-time domains is taught separately or in integrated form, the present organization of the book provides both pedagogical and practical advantages. A considerable part of the subject matter in signals and systems is on analysis techniques (such as solution methods in the time and frequency domains) which, although conceptually similar, use different tools.

Introducing the tools and applying them separately simplifies the structure of the course. Another advantage of the present organization is that the analyses of signals and systems in the continuous- and discrete-time domains can stand on their own (both conceptually and in terms of analysis tools). Each domain may be taught without requiring the other. Thus, for programs that are designed to offer a DSP course, the discrete-time part of the book will satisfy the prerequisites of such a course.

Each part begins with the introduction of signals and their models in the time domain. It then defines systems, linearity, and time invariance, along with examples. Time-domain solution methods, such as convolution and differential/difference equations, are presented next, followed by the transform domains. These are brought together in capstone chapters on the system function and frequency response. Chapter 10 on sampling provides a bridge between the continuous- and discrete-time domains.

Each chapter is made of sections and no subsections. Each section addresses a single discussion item, starting with the introduction of a topic, mathematical tools used to address that topic, the application of those tools, and one or two examples. To a large extent, therefore, each section is a learning unit and can provide the student with a concluding marker in learning the subject. In that sense the sections are modular and convenient for instruction. The modular organization of the book provides a direct approach and an effective tool for learning the fundamentals of signals and systems. As a vehicle for lectures, 5 to 10 essential sections may be covered in an hour, while others may be assigned as outside reading or homework.

Reference to other sections, figures, formulas, and other chapters is kept to a minimum. This provides easy and direct access to material, a feature much preferred by students and instructors. The modular structure of the chapters and sections also makes the book a convenient tool for instructional needs in a wide range of teaching scenarios at various levels of complexity.

Illustrative examples, end-of-chapter problems, and supplementary problems with solutions comprise other important components of the book. The book contains a total of nearly 475 examples, 175 problems with solutions, and 750 end-of-chapter problems. The examples and problems are of two types: (1) mathematical analyses and exercises that represent abstractions of engineering subjects and (2) contextual problems, such as those seen in electric circuits and devices, communication, and control. For the EE and CPE student these subjects provide a context to convey and develop fundamental concepts in signals and systems.

Examples from familiar signals and tangible systems in engineering can illustrate the utility of the relevant mathematical analysis. They can make the subject more attractive and generate motivation. In accordance with the above pedagogy, the book assumes that the reader is familiar with the operation of basic circuits and devices (such as passive *RLC* circuits and active circuits including dependent sources and operational amplifier models) and uses these to illustrate and reinforce the mathematical concepts. It also assumes familiarity with elementary trigonometric functions, complex numbers, differentiation/integration, and matrices. The Appendix at the end of the book can be used to refresh the reader's memory on electric circuits.

ORGANIZATION OF CHAPTERS

The detailed outline of the first part, covering signals and systems in the continuous-time domain, is as follows.

Chapter 1 introduces various signal types (such as those that are natural, societal, or human-made) and their models. It shows that, as functions of time, signals are specified by a wide set of parameters and characteristics (e.g., rate of change, time course, periodicity, and fine, coarse, and nested-loops structures). Time averages are discussed, along with some simple operations on signals.

Chapter 2 is on sinusoids and contains a review of basic trigonometry. The examples in this chapter employ simple sinusoids in illustrating some topics of practical interest such as phase and group delay, power, and more.

Chapter 3 introduces the definitions of linearity and time invariance. Examples teach the student how to test for these properties. This initial chapter is not intended to cover all properties of LTI systems, but only as much as is needed at this stage in such a course on signals and systems. More exposure will be provided throughout other parts of the book.

Chapter 4 discusses the time-domain solution of LTI systems by convolution. Convolution of a system's unit-impulse response with the input produces the system's response to that input. The chapter starts with convolution as a method of obtaining the response of a linear system. It uses the linearity and superposition properties to develop the convolution sum and integral. It then illustrates their evaluation by numerical, analytical, and graphical methods. The filtering property of convolution is explained next. The chapter also briefly touches on deconvolution. This latter concept is brought up in future chapters on solutions in the frequency domain.

Chapter 5 presents the time-domain solution of LTI systems by an examination of their describing differential equations in classical form. Parallels are drawn between the homogeneous and particular components of the total solution and the familiar components in the response of physical systems; that is, the natural and forced parts of the response from system analysis and design. The homogeneous and particular components of the total solution are then also related to the zero-input and zero-state responses. An example of a numerical computation of a response is provided at the end of the chapter.

Chapter 6 analyzes the solution of LTI systems by the Laplace transform in the frequency domain. Both the unilateral and bilateral forms of the transform are considered. The first half of the chapter focuses on the unilateral version, its inverse evaluation by the partial fraction expansion method, and some applications. The residue method of finding the inverse is also presented. The second half addresses the bilateral Laplace transform and its inverse. Comprehensive examples demonstrate how to obtain the response of an LTI system by the frequency domain approach and observe parallels with those in the time domain.

Chapters 7 and 8 are on Fourier analysis in the continuous-time domain. Chapter 7 discusses the Fourier series expansion of periodic signals, in both trigonometric and exponential forms, and visualizes methods of extending the expansion to nonperiodic signals, which is a topic presented in detail in Chapter 8. Introduction of the impulse function in the frequency domain provides a unified method of Fourier analysis for a large class of signals and systems. The convolution property of the Fourier transform enables system analysis in the frequency domain. The frequency variable f (or ω) in that analysis is more reminiscent of the actual real-world physical frequency than the complex frequency shown by s in the method of Laplace transforms.

Chapter 9 envisions a multiangle capstone perspective of the analysis methods presented up to this point. It introduces the system function, poles and zeros, the frequency response, and Bode plots. It then explains their relationships to each other and to the time-domain characteristics of a system. A vectorial interpretation of the system function and frequency response is included in order to help provide a qualitative understanding of the system's characteristics. Modeling a system by its dominant poles is then discussed in sufficient detail and illustrated by examples for first- and second-order systems. Interconnections between systems and the concept of feedback are then covered. Finally, the chapter concludes with a brief review of the effect of feedback on system behavior, along with an example of controller design.

The detailed outline of the second part of the book, covering signals and systems in the discrete-time domain, is as follows.

Chapter 10 is on the time-domain sampling of continuous-time signals and their reconstruction. It uses Fourier transform techniques and properties developed for continuous-time signals in Chapter 8 in order to derive the minimum sampling rate and a method for the error-free reconstruction of low-pass signals. The continuous-time signals used in the examples of this chapter are mostly built around 1 Hz to coincide, without loss of generality, with the *normalized* frequency encountered in the Fourier analysis of discrete-time signals in the second part of the book. The effects of sampling rate, the reconstruction filter, nearly low-pass signals, and the aliasing phenomenon are discussed. The chapter also extends the presentation to sampling and reconstruction of complex low-pass and bandpass signals.

Chapters 11 and 12 introduce discrete-time signals and LTI systems in a way that parallels the discussions in Chapters 1 and 3.

The discrete convolution and difference equations are discussed in Chapters 13 and 14. In these chapters, as in Part One, in addition to developing quantitative analytical techniques the text also aims to develop the student's intuitive sense of signals and systems.

Chapter 15 is on the z-transform and parallels the Laplace transform in providing a frequency domain analysis of discrete-time signals and systems. However, the chapter starts with the bilateral transform and its applications to signals and systems and then proceeds to the unilateral transform. The z-transform is normally defined on its own. However, it is related to the Laplace transform and can be derived from it. The relationship between these two transforms is explained in the chapter.

Chapters 16 and 17 discuss the discrete-time Fourier transform (DTFT) and the discrete Fourier transform (DFT). As is true of the z-transform, they can be defined on their own or be derived as extensions of the Fourier transform for continuous-time signals. Primary emphasis is given to the DTFT and DFT as stand-alone operations with a secondary reminder of their relationship to the Fourier transform for continuous-time signals. Having introduced the DTFT as an analysis tool, Chapter 16 introduces the concepts of decimation, interpolation, and sampling rate conversion. These concepts have a special place in discrete-time signal processing.

Chapter 18 is the discrete-time counterpart to Chapter 9. It encapsulates the system function, poles and zeros, and the frequency response. It includes an introduction to digital filters with relevant examples.

The book includes an appendix on the basics of electric circuit analysis.

PROJECTS

As a concept, projects cannot only reinforce a theory learned but also motivate it. Ideally, they have the most impact on learning when most of their formulation and solution steps are left to the student. With these ideas in mind, each chapter includes one or more projects germane to the subject of that chapter. These projects present self-contained theory and procedures that lead the student toward expected results and conclusions. Most projects are designed to be carried out in a laboratory with basic measurement instruments. They can also be implemented by using a simulation package such as Matlab. It is, however, recommended that they be done in a real-time laboratory environment whenever possible. For example, despite its simplicity, a simple passive *RLC* circuit can demonstrate many features of first- and second-order systems. Similarly, time and frequency responses, system function, oscillations, and the stability of systems can best be explored using an actual op-amp circuit.

PEDAGOGY

The book is designed with the following pedagogical features in mind.

1. One learns from being exposed to examples, each of which addresses a single, not-too-complicated question. The examples should be easy to grasp, relevant, and applicable to new scenarios.

2. One learns by doing, whether using paper and pencil, computer tools, projects, or laboratory experiments employing hardware. This leads students to search, explore, and seek new solutions. This point, along with the previous one, helps them develop their own methods of generalization, concept formation, and modeling.

3. One learns from exposure to a problem from several angles. This allows for the analysis of a case at various levels of complexity.

4. One needs to develop a qualitative and intuitive understanding of the principles behind, applications of, and solutions for the particular problem at hand. This is to supplement the quantitative and algorithmic method of solving the problem.

5. One benefits a great deal from gradual learning; starting from what has already been learned, one builds upon this foundation using familiar tools. In order to discuss a complex concept, one starts with the discussion and use of a simpler one upon which the former is based. An example would be introducing and using mathematical entities such as the frequency-domain variables s and z initially as complex numbers in exponential functions. The student first becomes familiar with the role the new variables play in the analysis of signals and systems before moving on to the Laplace and z-transforms. Another example would be the frequency response, a concept that can be developed within the existing realm of sinusoids and as an experimentally measurable characteristic of a system, as opposed to the more mathematical formulation of evaluating the system function on the imaginary axis of the complex plane. Yet another example would be the convolution integral, which can initially be introduced as a weighted averaging process.

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Northern California Earthquake Data Center and Berkeley Seismological Laboratory, University of California, Berkeley, www.ncedc.org.

Bureau of Labor Statistics, www.bls.gov.

ONLINE RESOURCES

INSTRUCTOR AND STUDENT WEBSITE

Available at www.mhhe.com/nahvi 1e are a number of additional instructor and student resources to accompany the text. These include solutions for end-of-chapter problems and lecture PowerPoints. The site also features COSMOS, a complete online solutions manual organization system that allows instructors to create custom homework, quizzes, and tests using end-of-chapter problems from the text.

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Chapter 1

Introduction to Signals

Contents

Introduction and Summary

A major part of this book is about signals and signal processing. In the conventional sense, signals are elements of communication, control, sensing, and actuation processes.They convey data, messages, and information from the source to the receiver and carry commands to influence the behavior of other systems. Radio, television, telephone, and computer communication systems use time-varying electromagnetic fields as signals. Command, control, and communication centers also use electromagnetic signals. Living systems employ sensory signals such as acoustic, visual, tactile, olfactory, or chemical. They also send signals by motion of their body parts such as the arms, hands, and face. The presence or unexpected absence of such signals is then detected by other living systems with whom communication is made. Neurons of the nervous system communicate with other neurons and control activity of muscles by electrical signals. Another group of signals of interest are those that represent variations of economic and societal phenomena (e.g., historical unemployment rate, stock market prices, and indexes such as the Dow Jones Industrial Average, median prices of houses, the federal funds interest rate, etc.). Still another group of signals of interest represent natural phenomena (pressure, temperature, and humidity recorded by weather stations, number of sunspots, etc.).

Signals, Information, and Meaning

As an element of communication and control processes, a signal is strongly related to other concepts such as data, codes, protocols, messages, information, and meaning. However, our discussion of signals and signal processing will be, to a large degree, confined outside of the context of such facets attached to a signal.

Signals and Waveforms

In this book a signal is a time-varying waveform. It may be an information-carrying element of a communication process that transmits a message. It may be the unwanted disturbance that interferes with communication and control processes, distorts the message, or introduces errors. It may represent observations of a physical system and our characterizations of it regardless of its influence (or lack thereof) on other systems.

We are interested in signals used in fields such as electrical communication, speech, computer and electronics, electromechanical systems, control systems, geophysical systems, and biomedical systems. Such signals represent variations of physical phenomena such as air pressure, electric field, light intensity and color in a visual field, vibrations in a liquid or solid, and so on. These signals are waveforms that depend on one or more variables such as time or space. (For example, a speech signal is a function of time but can also vary as a function of another variable such as space, if it is multiply recorded at several locations or if the microphone is moved around relative to the speaker. Geophysical signals are another set of such examples. Weather data collected at various stations at various times are still another such set.) The words *signals* and *waveforms* are, therefore, often used interchangeably.

Signals and Functions

We represent signals by mathematical functions. To this end, we often use the words *signals, waveforms*, and *functions* synonymously. Some simple elementary functions used in the mathematical representation of signals are steps, impulses, pulses, sinosoids, and exponentials. These are briefly described in section 1.14 of this chapter. Sinusoids are of special interest in signal analysis. They are treated in detail in the next chapter.

The chapter aims at achieving two interrelated goals. First, it presents the reader with a qualitative landscape of signals of common interest by giving actual examples such as natural, societal, financial, voice and speech, communication, and bioelectric signals. Second, in order to prepare the reader for the analytical conversation carried on throughout the book, it introduces, in detail, signal notations and elementary mathematical functions of interest (such as step, impulse, exponential, sinusoid, sinc, pulse, windows), and their basic properties such as the time average, even and odd parts, causality, and periodicity. It then introduces time transformation and scaling, which are parts of many mathematical operations on signals. Random signals are briefly introduced to broaden the scope of applications and projects. The Matlab programs in this chapter focus on generating and plotting signals and functions.

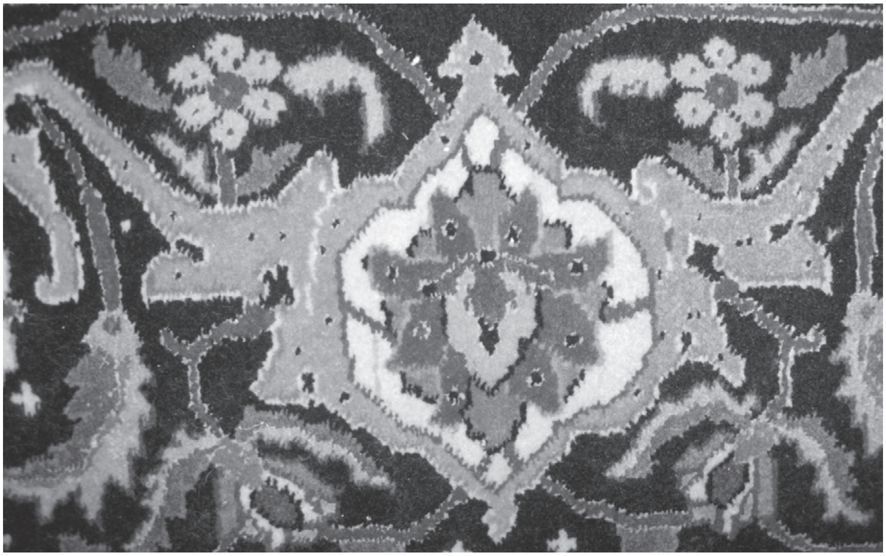
1.1 Discrete Versus Continuous; Digital Versus Analog

Discrete Versus Continuous

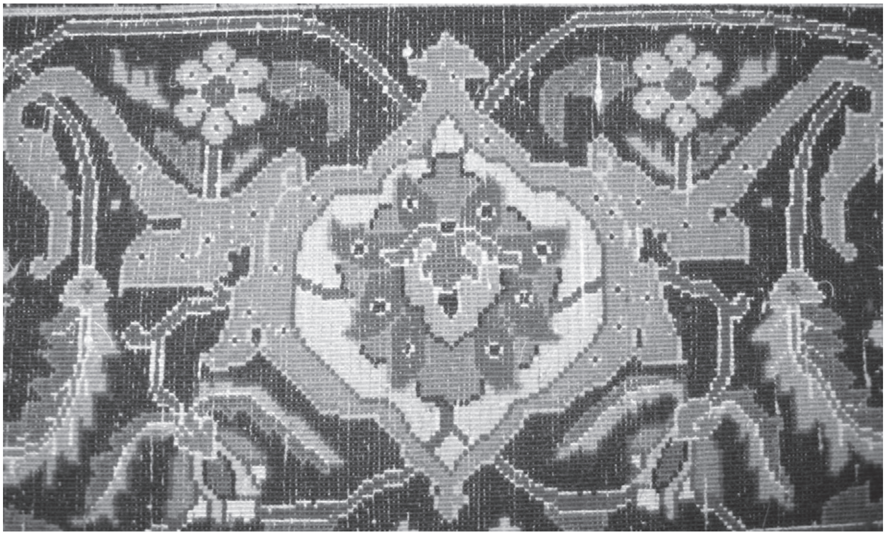
Some quantities appear to be analog in nature. They change in a continuous way. Geometrical and physical quantities are considered continuous. Some other examples are the following: time; muscle force; the intensity of sound, light, or color; the intensity of wind, ocean waves, or pain; the motion of the sun, moon, and planets; water flow from a spring; the growth of a tree; the radius of a circle; voltage, current, and field strength; the distance between two points; the size of a foot; the circumference of a waist measured using a rope.

Some quantities appear to be discrete and change in a discrete way. Quantities expressed by numbers are discrete. Some examples are the following: the number of fingers on our hands, teeth in our mouths, trees in an orchard, oranges on a tree, members of a tribe; the number of planets and stars in the sky; the price of a loaf of bread; the rings on a tree; the distance between two cities; the size of a shoe; voltage, current, and weight measurements; a credit card monthly balance; waist size measured using a pinched belt.

On some occasions a discrete quantity is treated (modeled) as a continuous variable. This may be for modeling convenience (e.g., the amount of hair on a person's head) or an effect of perception (e.g., the construction of a carpet, see Plate 1.1). Similarly, a continuous quantity may be expressed in discrete form. The number of colors appearing on a computer monitor is an example of this. Once a continuous variable is measured and expressed by a number it becomes discrete. Most computations are done in the discrete domain.



(*a*) Front side of a segment of a carpet appears as having a continuous structure.



(*b*) The back side shows the discrete structure.

Plate 1.1 An example of continuous versus discrete representation of a signal can be observed by comparing the continuous front *(a)* with the discrete back *(b)* of a hand-woven carpet. The carpet has a discrete structure that characterizes the carpet by the number of knots per unit length, measured on the back side. The pattern shown in this plate is repeated several times throughout the carpet (not shown). The pattern is deterministic and is provided to the weaver for exact implementation. The weaver, however, introduces unintentional randomness seen in the product as slight variations. These variations are not observed by an untrained person but are detected by a specialist or through magnification.

Digital Versus Analog

A continuous-time signal is converted into a discrete signal by sampling. The samples, however, are analog because they assume a continuous range of values. We can convert the sample values into a discrete set by assigning the value of each sample to one of n predetermined levels. The result is a digital signal. For instance, in the case of a binary discrete signal, there are only two predetermined levels into which the analog samples are forced: 0 and 1. Changes between these levels occur at the arrival time of a clock signal. Because of finite wordlength, which determines the resolution in the magnitude value of discrete-time functions, these are often called digital signals and discrete systems are called digital systems.

1.2 Deterministic Versus Random

Signals are said to be deterministic or probabilistic (random). Once it appears, a deterministic signal does not provide any new information, unless some of its properties change with time. A signal that could be predicted from its past values causes less surprise and carries less information. The only information available from the 60 Hz sinusoidal signal of a power line is its presence.[1] In contrast, a code that reduces the correlation between consecutive segments of the signal increases the information content of the signal.[2]

Some signals originating from natural, living, or societal systems vary with time in an exact and regular way, making them predictable. An example would be the rising sun. Or take the regularity of an electrocardiogram signal (EKG) that conveys health information. The appearance of an irregularity is taken as a sign of disease. As a third example, consider an advertisement for a candy brand touting the consistency of the product. In this case, the information intended to be conveyed is predictability. Within the above category we may also include signals that vary somewhat in a regular and complicated (but not random) manner. An example would be the positions of the planets in the sky, perceived and determined by an observer of the sky 5,000 years ago, and their application in predicting future events and fortune telling by astronomers, astrologers, and seers, or in decision making by rulers or elected officials in the past or current times.

In contrast, a signal may contain some stochastic (random) characteristic contained within quasi-deterministic features and, depending on the degree of randomness in the signal, still be considered predictable probabilistically within some statistical error rate.[3] The combination of regularity and randomness in natural or societal signals is to be expected. Such signals are the collective result of many interacting elements in physical systems. The signals, therefore, reflect the regularity of the physical structure and the irregularity of the message. The apparent randomness may also be due to our lack of

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[1] The information provided by the above signal is only one *bit*. However, the information is normally very valuable and important.

[2] By some definitions, signals that appear most random contain the most information.

[3] Sometimes the signal might appear to be totally unpredictable (e.g., appearance and time of a shooting star).

knowledge about the system responsible for generating the signal or an inability to incorporate such knowledge in a model.

1.3 Examples of Natural and Societal Signals

Sunspot Numbers

Of interest in electrical communication (as well as in other fields) is the level of solar activities, signaled by the number of sunspots as a function of time. Figure 1.1(*a*) shows the annual mean number of sunspot records from AD 1700 to 2010 with the abscissa indicating days. A clear feature of the record is the pattern of its variation, which exhibits 28 cycles of activity with an average period of 11 years during the past 310 years. Each cycle has its own duration, peak, and valley values. One can also observe some waxing and waning of the peak values, suggesting stratification of the record into three centennial groups of cycles (one segment consisting of cycles from 1770 to about 1810, the second segment from 1810 to 1900, and the third from 1900 to 2010). In relation to the signal of Figure 1.1(*a*) one may define several variables exhibiting random behavior. Examples of such random variables are the number of sunspots (daily, monthly, and yearly numbers), period of cyclic variations, peaks and valleys, and rise and fall times within each cycle. A first step in the analysis of signals such as that in Figure 1.1(*a*) would be to estimate the mean and variance of the variables. To acquire more insight one would also find the correlation and interdependencies between them. Toward that goal one would use a more detailed set of sunspot data such as average daily measurements. These provide a better source for analysis of cyclic variation and fine structures within a cycle. Figure 1.1(*b*) plots such a set of data for the period of January 1, 1818, through September 30, 2010. In this figure and its subsets shown in Figure 1.1(*c*), (*d*), and (*e*), the numbers on the abscissas indicate the day, counting from the beginning of the time period for that figure. The dates of the beginning and end of the time period are shown at the left and right sides of the abscissa, respectively. The data of Figure 1.1(*b*) show many days with average sunspot numbers above 200 and even some above 300 per day. Daily averages also provide a better tool for analysis of cyclic variation and fine structure within a cycle. Bursts of activities lasting 10 to 20 days are observed in Figure 1.1(*c*), which plots the data for the years January 1, 1996, through December 31, 2009, covering the most recent cycle. Two extreme examples of yearlong daily measurements during the most recent cycle of sunspot activities are shown in Figure 1.1(*d*) and (*e*). Figure 1.1(*d*) (for the year 2001, which was an active one) indicates high levels of activity with the occurrence of strong periodic bursts. In contrast, Figure 1.1(*e*) (for the year 2009) shows weak levels of activity but still occurring in the form of bursts.

Atmospheric CO_2 Content

Carbon dioxide (CO_2) is one of the greenhouse gases associated with thermal changes of the atmosphere and is a signal for it. Monitoring atmospheric CO_2 and trends in its temporal and spatial variations is of potential importance to every person. Scientific work on atmospheric carbon dioxide uses long-term historical information as well as

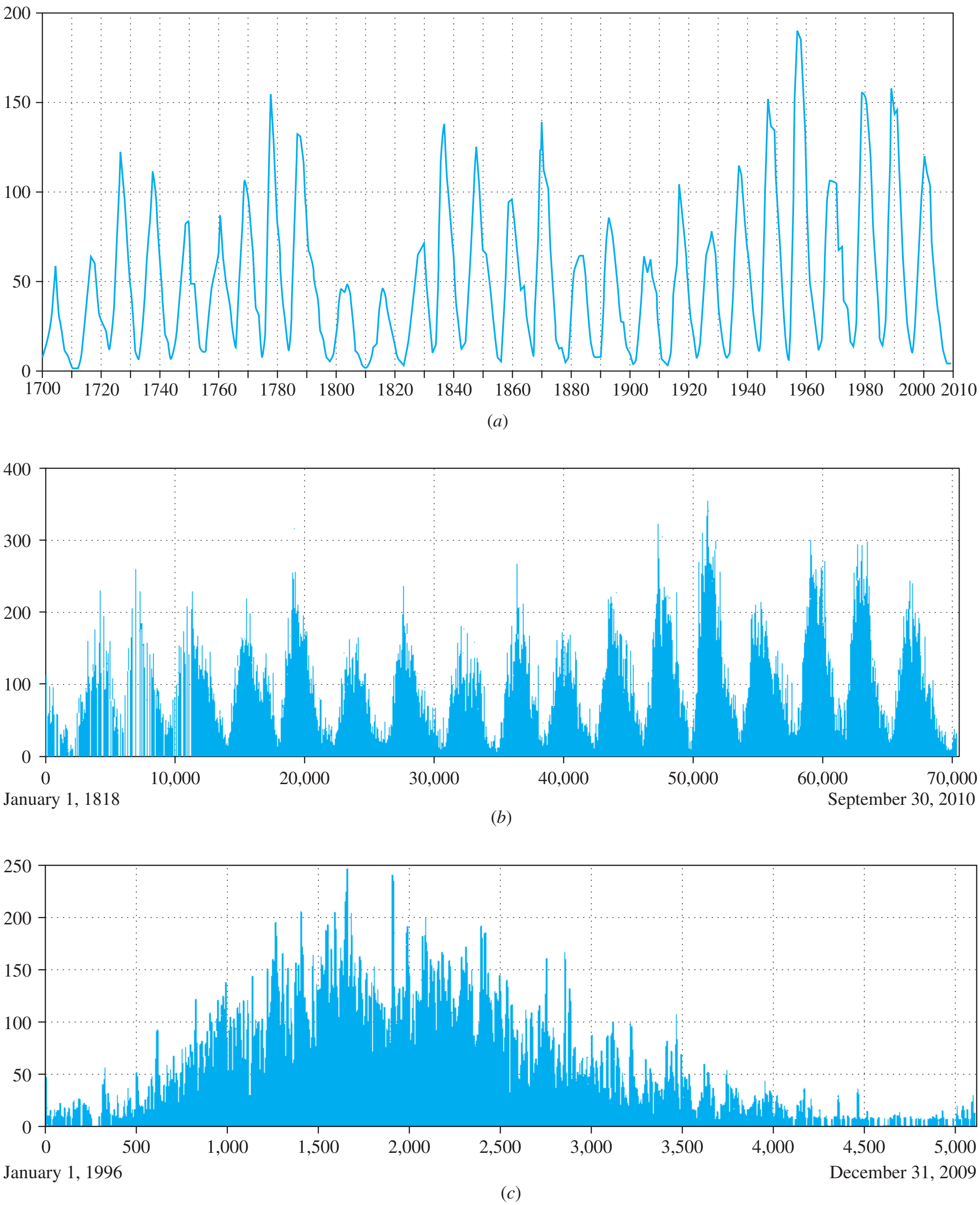
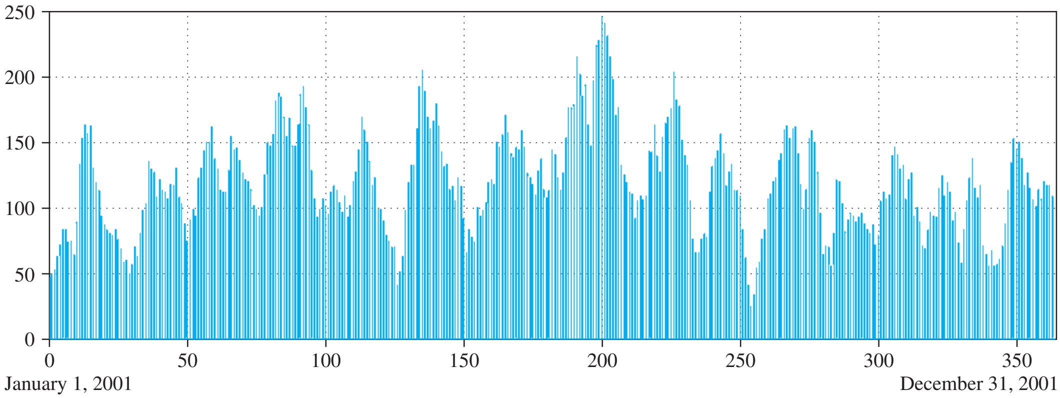
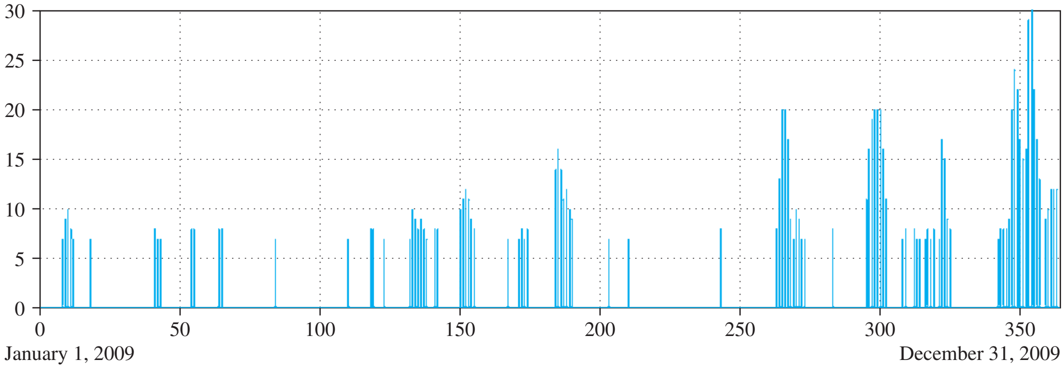


FIGURE 1.1 Sunspot numbers. *(a)* Mean annual values (AD 1700–2010); *(b)*, *(c)*, *(d)*, and *(e)* daily values for selected time intervals during 1818 to 2010.

Source: National Oceanic and Atmospheric Association's (NOAA) National Geophysical Data Center (NGDC) at www.ngdc.noaa.gov.



(d)



(e)

FIGURE 1.1 (*Continued*)

contemporary observations. Historical records indicate that trends in atmospheric CO_2 are associated with glacial cycles. During the last 400,000 years, the CO_2 content of the atmosphere fluctuated between a value below 200 to nearly 300 ppmv (parts per million volume). The data are obtained by analyzing gas contents of the air bubbles entrapped in polar ice sheets.

An example of the historical data is shown in Figure 1.2(*a*) which plots the results of measuring the CO_2 content of air bubbles in the ice cores of Vostock station in Antarctica. The air in these bubbles is from 400,000 to 5,000 years ago. The data for the plot of Figure 1.2(*a*) show long-term cyclic variations of 80 to 120,000 years with minima and maxima of nearly 180 and 300 ppm, respectively. Present-day atmospheric CO_2 shows much higher values which are unprecedented during the past half a million years. Contemporary measurements are done under controlled and calibrated conditions to avoid the influence of local sources (such as emissions) or environmental elements (such as trees) that absorb, trap, or remove CO_2 from the air. Figure 1.2(*b*) plots contemporary data for 1959 to 2010 from measurements at the Mauna Loa observatory station in Hawaii (chosen for its suitable location in terms of providing base measurements). The

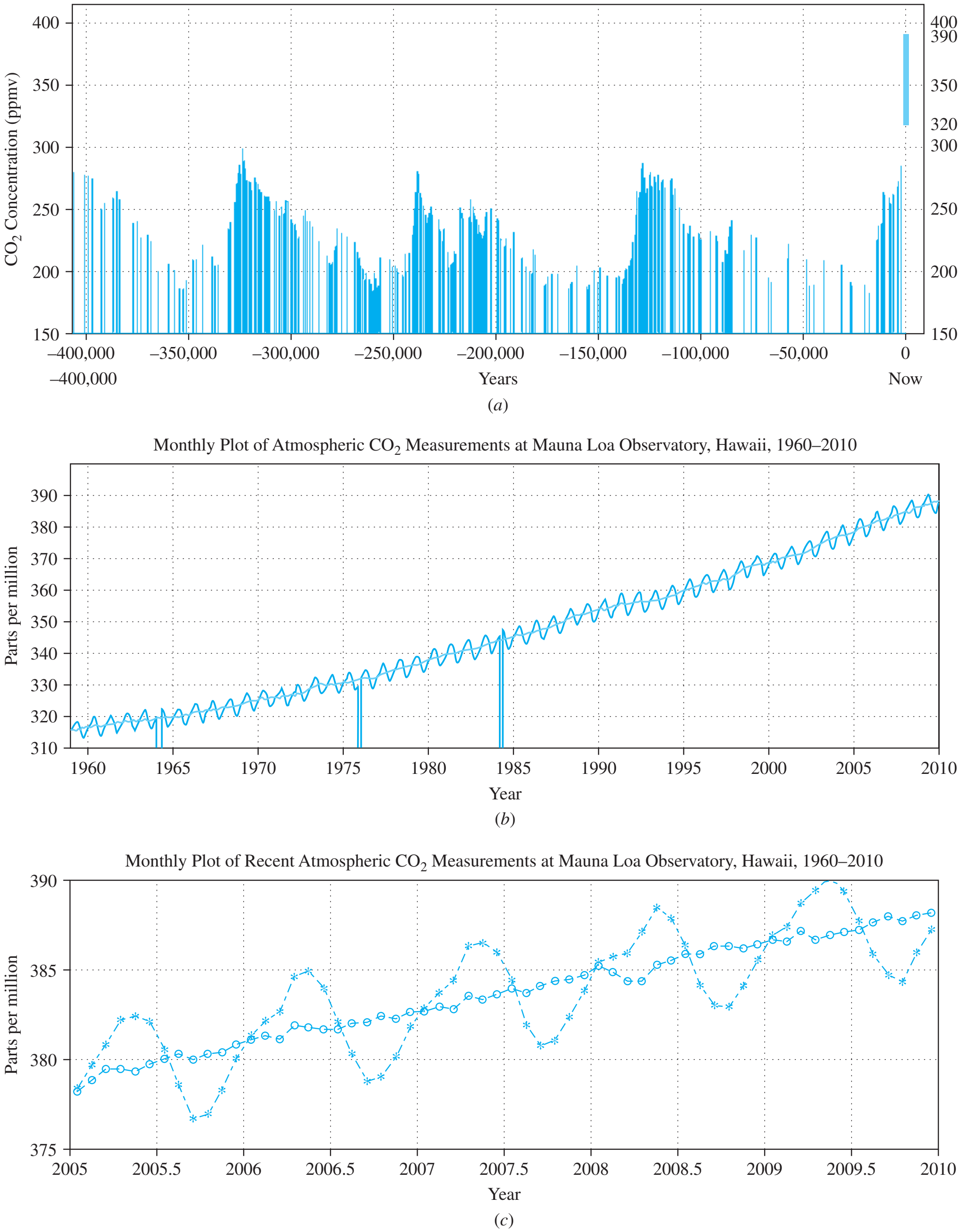


FIGURE 1.2 *(a)* Historical carbon dioxide record from Vostok ice cores. *(b)* and *(c)* Monthly carbon dioxide record from Mauna Loa observatory, Hawaii. The record for 1960–2010 is shown in *(b)*, while *(c)* shows the record for 2005–2010 at a higher resolution.

Sources: *(a)* Carbon Dioxide Information Analysis Center (CDIAC) of the U.S. Department of Energy at http://cdiac.ornl.gov/. *(b)* and *(c)* Dr. Pieter Tans, NOAA/ESRL (www.esrl.noaa.gov/gmd/ccgg/trends/).